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# Engineering Notes

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## Equilibrium Control of Electrodynamic Tethered Satellite Systems in Inclined Orbits

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### Introduction

SINCE the idea of the space tether system was suggested, much work has been done in this field. A main concern of tether research is control of the motion of the tether. Pasca [1] investigated the control of in-plane transversal vibrations of a tethered satellite system using a longitudinal thrust force. Two nonlinear control laws were developed to stabilize the station-keeping phase of the tether system. Brian and Jordi [2] developed a linear control system to control the unstable mode of an atmospheric tether system, with the tether modeled as a rigid rod, by using the attachment point motion and thrust as inputs. Yu [3] investigated a dynamic model for control of a mass-distributed and extensible tether system by considering the stationary configuration of the system. Tani and Qiu [4] proposed a two-dimensional motion control of the tethered satellite by using electrodynamic force generated through the interaction between the geomagnetic field and an applied electric current passing through the tether. Pelaez and Lorenzini [5] proposed two control schemes by using electrodynamic force to convert an unstable periodic orbit of the system into an asymptotically stable one. Williams et al. [6] developed a control method by using both tension and electromagnetic forces to regulate the librational motion of the tethered satellite system. Mankala and Agrawal [7] suggested a feedback control law for implementation of the in-plane tether maneuvers from one equilibrium configuration to another by means of electrodynamic force which is generated by the interaction between tether current and geomagnetic field.

The stability control of equilibrium positions is an important technical problem. Steiner et al. [8] proposed a method to control the in-plane oscillation about the radial equilibrium positions by adjusting the tether tension with a tether modeled as a massless and rigid one. However, Steiner [8] only considered the in-plane motion of the tethered satellite system. In this paper, a more complete model of a tethered satellite system is considered, in which a main satellite and a subsatellite is connected by a conductive tether with mass distributed along it. To regulate both the in-plane and out-of-plane motions of the tether, the current and the rate of change in tether length are employed as two control parameters. A feedback control law is proposed to maintain the radial equilibrium position of the system. It is found that this control law is not applicable for the equatorial plane because no out-of-plane force is available there. For each inclined orbit, it is shown that there are two singularity points. To avoid these points, and by considering some other practical restrictions, the proposed control law is divided into four conditional parts. Numerical examples are provided in this Note and the results validate the applicability of the proposed control law.

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### Equations of Motion

Consider a tethered satellite system moving around Earth in the geomagnetic field. The main satellite $S_1$ with mass $m_1$ is restricted to a certain orbit. A tether with variable length $L$ is attached to the main satellite and can rotate freely around the satellite in any direction. A subsatellite $S_2$ with mass $m_2$ is attached to the other end of the tether. The tether is made of thin, strong, and uniform material with mass distributed along the length of the tether. The transverse and longitudinal oscillations of the tether are assumed to be negligible. The configuration of the system is shown in Fig. 1.

The kinetic and gravitational potential energies of the tether are given by

$$\begin{aligned} T_t = {} & \tfrac{1}{2}\rho L\{\dot r^2 + r^2\dot\theta^2 + \dot L^2 + \tfrac{1}{3}L^2[\dot\beta^2 + (\dot\theta+\dot\alpha)^2C^2\beta] \\ & + 2\dot L C\beta(\dot r C\alpha + r\dot\theta S\alpha) - L[(\dot\theta+\dot\alpha)C\beta(\dot r S\alpha - r\dot\theta C\alpha) \\ & + \dot\beta S\beta(\dot r C\alpha + r\dot\theta S\alpha)]\} \end{aligned} \tag{1}$$

$$\begin{aligned} V_t &= -\mu\rho\int_0^L \frac{1}{\sqrt{\xi^2 + r^2 + 2\xi r C\alpha C\beta}}\,\mathrm{d}\xi \\ &= -\mu\rho\,\ell_n\frac{rC\alpha C\beta + L + \sqrt{r^2 + L^2 + 2rLC\alpha C\beta}}{rC\alpha C\beta + r} \end{aligned} \tag{2}$$

where $\rho$ is the mass per unit length of the tether and $\mu$ is the gravitational constant of Earth, taken as $3.986\times10^5\ \mathrm{km}^3/\mathrm{s}^2$. Here and in subsequent equations, sin and cos are abbreviated as $S$ and $C$, respectively. The kinetic and gravitational potential energies of the tethered satellite system are given by

$$T = \tfrac{1}{2}[m_1 + \rho(L_0 - L)](\dot r^2 + r^2\dot\theta^2) + T_t + T_2 \tag{3}$$

$$V = -\frac{\mu[m_1 + \rho(L_0 - L)]}{r} + V_t + V_2 \tag{4}$$

where $L_0$ is the initial length of the tether; $T_2$ and $V_2$ are the kinetic and potential energies of the subsatellite $S_2$, given by

$$\begin{aligned} T_2 = {} & \tfrac{1}{2}m_2\{\dot r^2 + r^2\dot\theta^2 + \dot L^2 + L^2[\dot\beta^2 + (\dot\theta+\dot\alpha)^2C^2\beta] \\ & + 2\dot L C\beta[\dot r C\alpha + r\dot\theta S\alpha] - 2L[(\dot\theta+\dot\alpha)C\beta(\dot\alpha S\alpha - r\dot\theta C\alpha) \\ & + \dot\beta S\beta(\dot r C\alpha + r\dot\theta S\alpha)]\} \end{aligned} \tag{5}$$

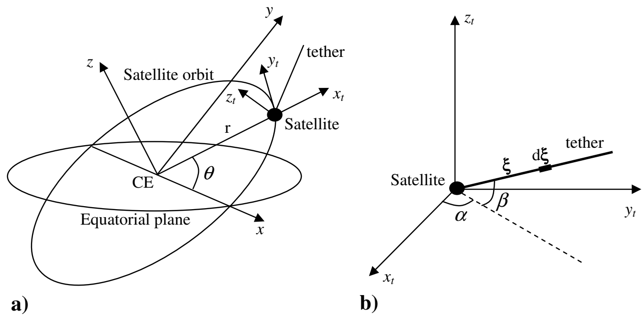


**Fig. 1 Tether-satellite system: a) orbital motion in inertial coordinates fixed to the center of the Earth; b) tether librational motion with respect to the main satellite.**

$$V_2 = -\frac{\mu m_2}{\sqrt{L^2 + r^2 + 2LrC\alpha C\beta}} \tag{6}$$

Using Lagrange's method, the equations of motion corresponding to $\alpha$ and $\beta$ can be written as

$$\begin{aligned}&(\rho L + 2m_2)L[\dot{L}(\dot{\theta} + \dot{\alpha})C^2\beta - \tfrac{1}{2}\ddot{r}S\alpha C\beta + \dot{r}\,\dot{\theta}\,C\alpha C\beta \\ &\quad + \tfrac{1}{2}r\ddot{\theta}C\alpha C\beta + \tfrac{1}{2}r\dot{\theta}^2 S\alpha C\beta] \\ &\quad + (\rho L + 3m_2)L^2[\tfrac{1}{3}(\ddot{\theta} + \ddot{\alpha})C^2\beta - \tfrac{2}{3}(\dot{\alpha} + \dot{\theta})\dot{\beta}S\beta C\beta] \\ &\quad + V_\alpha = Q_\alpha\end{aligned} \tag{7}$$

$$\begin{aligned}&(\rho L + 2m_2)L[\dot{L}\,\dot{\beta} - \tfrac{1}{2}\ddot{r}C\alpha S\beta - \dot{r}\,\dot{\theta}\,S\alpha S\beta - \tfrac{1}{3}r\ddot{\theta}S\alpha S\beta + \tfrac{1}{2}r\dot{\theta}^2 C\alpha S\beta] \\ &\quad + \tfrac{1}{3}(\rho L + 3m_2)L^2[\ddot{\beta} + (\dot{\alpha} + \dot{\theta})^2 S\beta C\beta] + V_\beta = Q_\beta\end{aligned} \tag{8}$$

where $Q_\alpha$ and $Q_\beta$ are the generalized forces corresponding to $\alpha$ and $\beta$, respectively; $V_\alpha$ and $V_\beta$ indicate $\partial V/\partial\alpha$ and $\partial V/\partial\beta$, respectively. Since $L \ll r$, using Taylor's expansion, approximate expressions for $V_\alpha$ and $V_\beta$ are given by

$$V_\alpha \approx -\frac{\mu\rho L^2 S\alpha C\beta}{2(r^2 + L^2 + 2rLC\alpha C\beta)} - \frac{\mu m_2 LrS\alpha C\beta}{\sqrt{L^2 + r^2 + 2LrC\alpha C\beta}^3} \tag{9}$$

$$V_\beta \approx -\frac{\mu\rho L^2 C\alpha S\beta}{2(r^2 + L^2 + 2rLC\alpha C\beta)} - \frac{\mu m_2 LrC\alpha S\beta}{\sqrt{L^2 + r^2 + 2LrC\alpha C\beta}^3} \tag{10}$$

When current passes through a conductive tether in the geomagnetic field, a Lorentz force is generated. In this Note, the geomagnetic field is modeled as a nontilted, Earth center dipole [7]. By simple coordinate transformations, the magnetic field can be expressed in the coordinate system $x_t y_t z_t$ (see Fig. 1) as

$$\mathbf{B} = -\frac{3\mu_0 M_{\mathrm{dp}} SiS\theta}{4\pi r^3}\mathbf{i_t} + \frac{\mu_0 M_{\mathrm{dp}} SiC\theta}{4\pi r^3}\mathbf{j_t} + \frac{\mu_0 M_{\mathrm{dp}} Ci}{4\pi r^3}\mathbf{k_t} \tag{11}$$

where $\mu_0 = 4\pi \times 10^{-7}$ Tm/A is the permeability of space, $M_{\mathrm{dp}} = 7.788 \times 10^{22}$ Am$^2$ is the dipole strength. Note that the magnitude of the magnetic field is of the order $1/r^3$. Therefore, we assume that the magnetic field remains constant along the length of the tether. The Lorentz force $\mathbf{dF}$ acting on an element d$\xi$ of the tether is given by

$$\mathbf{dF} = I\mathbf{d}\xi \times \mathbf{B} \tag{12}$$

where $I$ is the current passing through the tether and $\mathbf{d}\xi$ is the vector of magnitude d$\xi$ in the direction of current, given by

$$\mathbf{d}\,\xi = \mathrm{d}\xi(C\alpha C\beta\mathbf{i_t} + S\alpha C\beta\mathbf{j_t} + S\beta\mathbf{k_t}) \tag{13}$$

Using the virtual work done by the electrodynamic force, the generalized forces corresponding to $\alpha$ and $\beta$ can be written as

$$Q_\alpha = \frac{\mu_0 M_{\mathrm{dp}} IL^2}{8\pi r^3}\Psi_1 \tag{14}$$

$$Q_\beta = \frac{\mu_0 M_{\mathrm{dp}} IL^2}{8\pi r^3}\Psi_2 \tag{15}$$

where

$$\Psi_1 = C\beta(SiC\theta S\alpha S\beta - 2SiS\theta C\alpha S\beta - CiC\beta) \tag{16}$$

$$\Psi_2 = Si(C\theta C\alpha + 2S\theta S\alpha) \tag{17}$$

## Feedback Control Law

In this Note, the main satellite is restricted to a circular orbit so that $\dot{r} = \ddot{r} = \ddot{\theta} = 0$. Thus Eqs. (7) and (8) can be simplified as

$$\begin{aligned}&(\rho L + 2m_2)L[\dot{L}(\omega + \dot{\alpha})C^2\beta + \tfrac{1}{2}r\omega^2 S\alpha C\beta] \\ &\quad + \tfrac{1}{3}(\rho L + 3m_2)L^2[\ddot{\alpha}C^2\beta - 2(\dot{\alpha} + \omega)\dot{\beta}S\beta C\beta] \\ &\quad + V_\alpha = Q_\alpha\end{aligned} \tag{18}$$

$$\begin{aligned}&(\rho L + 2m_2)L[\dot{L}\,\dot{\beta} + \tfrac{1}{2}r\omega^2 C\alpha S\beta] \\ &\quad + \tfrac{1}{3}(\rho L + 3m_2)L^2[\ddot{\beta} + (\dot{\alpha} + \omega)^2 S\beta C\beta] \\ &\quad + V_\beta = Q_\beta\end{aligned} \tag{19}$$

where $\omega$ indicates the orbital angular velocity $\dot{\theta}$ of the main satellite. From Eqs. (18) and (19) it follows that the radial position $\alpha = 0$, $\beta = 0$ is a natural equilibrium position of the tether when the rate of change in tether length is $\dot{L} = 0$ and the current carried by the tether is $I = 0$. Incidentally, when the orbit of the main satellite is restricted to the equatorial plane, arbitrary in-plane equilibrium positions related to the current $I$ can be derived from Eqs. (18) and (19), which is similar to the results provided by Mankala and Agrawal [7]. However, when the orbit is an inclined one, because the generalized forces of both $\alpha$ and $\beta$, given by Eqs. (14) and (15) are time dependent, no static equilibrium positions can be found as long as the current does not equal zero.

A feedback control law is developed to regain the radial equilibrium position when oscillations around the equilibrium position occur due to perturbations acting on the tether. The rate of change in tether length $\dot{L}$ and the current carried by the tether $I$ are employed as two control parameters. A feedback linearized system can be achieved by considering the following control law:

$$\dot{L} = \frac{\Psi_2 B_1 - \Psi_1 B_2}{\Psi_1 A_2 - \Psi_2 A_1} \tag{20}$$

$$I = \frac{8\pi r^3}{\mu_0 M_{\mathrm{dp}} L^2}\frac{B_1 A_2 - B_2 A_1}{\Psi_1 A_2 - \Psi_2 A_1} \tag{21}$$

where

$$A_1 = (\rho L + 2m_2)L(\omega + \dot{\alpha})C^2\beta \tag{22}$$

$$A_2 = (\rho L + 2m_2)L\dot{\beta} \tag{23}$$

$$\begin{aligned}B_1 &= \frac{1}{3}(\rho L + 3m_2)L^2 \\ &\quad \times \left[\frac{1}{T_1^2}(\alpha_{\mathrm{ref}} - 2T_1\zeta_1\dot{\alpha} - \alpha)C^2\beta - 2(\dot{\alpha} + \omega)\dot{\beta}S\beta C\beta\right] \\ &\quad + \frac{1}{2}(\rho L + 2m_2)Lr\omega^2 S\alpha C\beta + V_\alpha\end{aligned} \tag{24}$$

$$B_2 = \frac{1}{3}(\rho L + 3m_2)L^2 \times \left[\frac{L}{T_2^2}(\beta_{\mathrm{ref}} - 2T_2\zeta_2\dot{\beta} - \beta) + (\dot{\alpha} + \omega)^2 S\beta C\beta\right] + \frac{1}{2}(\rho L + 2m_2)Lr\omega^2 C\alpha S\beta + V_\beta \tag{25}$$

Substituting Eqs. (20) and (21), into Eqs. (18) and (19), the closed-loop dynamics of $\alpha$ and $\beta$ become two second-order systems, given by

$$\ddot{\alpha} = \frac{1}{T_1^2}(\alpha_{\mathrm{ref}} - 2\zeta_1 T_1\dot{\alpha} - \alpha) \tag{26}$$

$$\ddot{\beta} = \frac{1}{T_2^2}(\beta_{\mathrm{ref}} - 2\zeta_2 T_2\dot{\beta} - \beta) \tag{27}$$

where $T_1$ and $T_2$ are the characteristic times of the second-order systems of $\alpha$ and $\beta$, both taken as 500 s; $\zeta_1$ and $\zeta_2$ are the damping coefficients, both taken as 1.1; $\alpha_{\mathrm{ref}}$ and $\beta_{\mathrm{ref}}$ are the reference values of $\alpha$ and $\beta$, respectively. For the problem of equilibrium control, $\alpha_{\mathrm{ref}} = \beta_{\mathrm{ref}} = 0$.

Note that when $\dot{\beta} \to 0$, the out-of-plane dynamics, given by Eq. (19), is independent of the rate of change in tether length. Therefore, for a problem of stability control around a radial equilibrium position, the control force of the out-of-plane angle is mainly provided by the electrodynamic force. When the orbit of the main satellite is restricted to the equatorial plane, however, the geomagnetic field is perpendicular to the orbital plane and thus no electrodynamic force in the direction of $\beta$ is available. This fact indicates that the proposed feedback control law is not applicable for the equatorial orbit. For an orbit with small orbital inclination angle, the in-plane component of the geomagnetic field, which is the substantial source of the out-of-plane electrodynamic force, is very small. Even though the proposed control law is applicable theoretically, the required current carried by the tether is expectedly very large, which will result in difficulties in practical application of the control law. Consequently, there is a threshold value for the orbit inclination angle so that the control cost is within a desirable range.

For a nonequatorial orbit, when the orbital angle $\theta$ equals 90 deg or 270 deg, the in-plane component of the geomagnetic field is in the direction of $\mathbf{i_t}$ according to Eq. (11). For the radial equilibrium position, there is again no out-of-plane electrodynamic force acting on the tether according to Eq. (12). Therefore, there are two singular points, $\theta = 90$ deg and $\theta = 270$ deg, in all the inclined orbits for the proposed control law. Given that $\dot{L}$ and $I$ equal zero, numerical simulations show that both $\alpha$ and $\beta$ will oscillate with amplitudes close to constant. Hence, to avoid these singular points, the simplest way is to let $\dot{L}$ and $I$ be zero at these points. However, considering the practical situations, to avoid a sudden increase in $\dot{L}$, the following control law of $\dot{L}$ and $I$ is employed when $\theta$ is within the range of [75 deg, 105 deg] and [255 deg, 285 deg]:

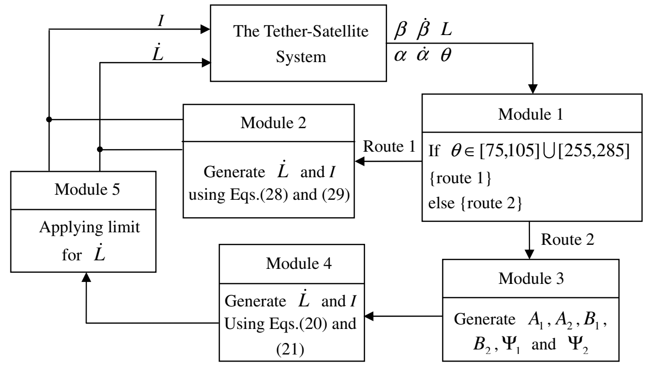


**Fig. 2 The structure of the closed-loop control system.**

where $\dot{L}_0$, $I_0$, and $t_0$ are the values of the rate of change in the tether length, the current, and the system time, respectively, when $\theta$ equals 75 deg or 255 deg; $\zeta_3$ and $T_3$ are taken as 1.1 and $\pi/(54\omega)$, respectively, so that $\dot{L}$ and $I$ become zero during these periods. In addition, when it comes to the moment that $\theta$ leaves these periods, it is possible that there is another sudden increase in $\dot{L}$ due to the shift between the control laws. To avoid the possible sudden increase in $\dot{L}$, a restriction for $\dot{L}$ is applied to the control law: once $\dot{L}$ given by Eq. (20) is larger than zero, the control law becomes

$$\dot{L} = 0 \tag{30}$$

$$I = 0 \tag{31}$$

Figure 2 provides an overview of the structure of the conditional feedback control law. In practical use, the current is provided by the on-board power source, which can be recharged by solar energy. The rate of change in tether length is controlled by a reel on which the tether coils.

## Simulation Results

To validate the proposed control law, numerical simulations are provided in this section. The initial tether length $L_0$ and the subsatellite's mass $m_2$ are taken as 1 km and 5 kg, respectively. The radius of orbit is 6600 km. As the first example, consider a main satellite orbit with orbit inclination angle of 45 deg. The system is under the influence of perturbations which result in the initial deviations 10 deg from radial equilibrium position for both $\alpha$ and $\beta$. Under such perturbations, the nonlinearities are not negligible and the utility of the proposed feedback linearization control law can be validated. Figure 3 shows the changes of $\alpha$ a) (solid line), $\beta$ a) (dashed line), $\dot{L}$ b) and $I$ c) as functions of time. Note that the in-plane angle $\alpha$, the out-of-plane angle $\beta$, the rate of change of tether length $\dot{L}$, and the current $I$ become zero after about $1 \times 10^4$ s, which indicates that the system returns back to the radial equilibrium position. During the whole process, the maximum values of $\dot{L}$ and $I$ are $-0.55$ m/s and 0.8 A, respectively, and are quite reasonable. Each pair of vertical dotted lines indicates the region where the orbit angle $\theta$ is within the range of [75 deg, 105 deg] or [255 deg, 285 deg]. Note that between the pair of vertical dotted lines, the rate of change

$$\dot{L} = \dot{L}_0\left(\frac{\zeta_3 + \sqrt{\zeta_3^2 - 1}}{2\sqrt{\zeta_3^2 - 1}} e^{\frac{-\zeta_3 + \sqrt{\zeta_3^2 - 1}}{T_3}(t - t_0)} + \frac{-\zeta_3 + \sqrt{\zeta_3^2 - 1}}{2\sqrt{\zeta_3^2 - 1}} e^{\frac{-\zeta_3 - \sqrt{\zeta_3^2 - 1}}{T_3}(t - t_0)}\right) \tag{28}$$

$$I = I_0\left(\frac{\zeta_3 + \sqrt{\zeta_3^2 - 1}}{2\sqrt{\zeta_3^2 - 1}} e^{\frac{-\zeta_3 + \sqrt{\zeta_3^2 - 1}}{T_3}(t - t_0)} + \frac{-\zeta_3 + \sqrt{\zeta_3^2 - 1}}{2\sqrt{\zeta_3^2 - 1}} e^{\frac{-\zeta_3 - \sqrt{\zeta_3^2 - 1}}{T_3}(t - t_0)}\right) \tag{29}$$

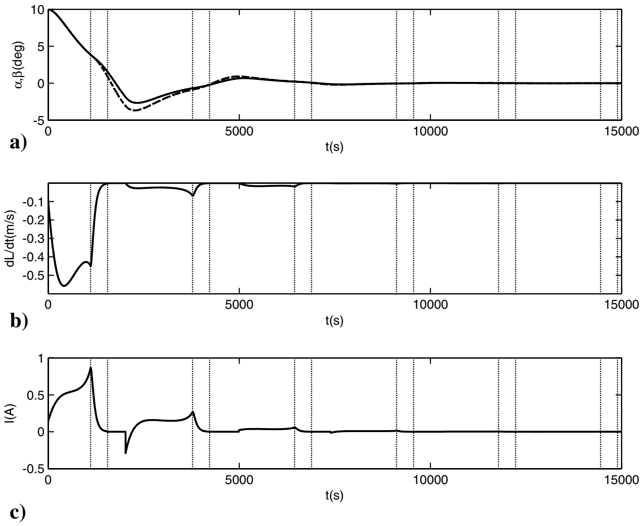


**Fig. 3 Changes of $\alpha$ a) (solid line), $\beta$ a) (dotted line), $\dot{L}$ b), and $I$ c) as functions of time when $i = 45$ deg.**

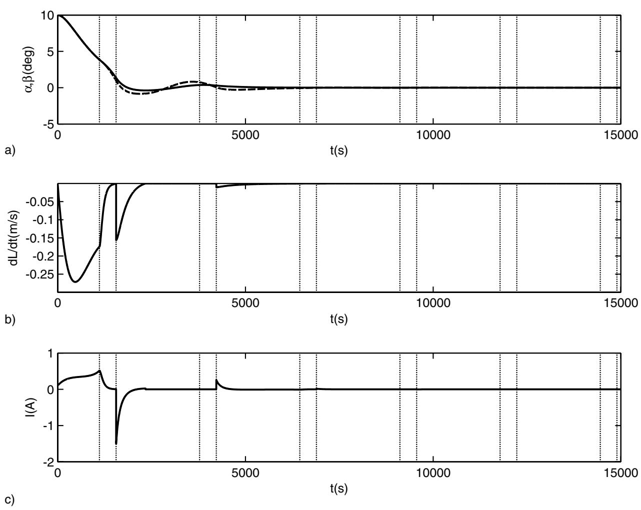


**Fig. 4 Changes of $\alpha$ a) (solid line), $\beta$ a) (dotted line), $\dot{L}$ b), and $I$ c) as functions of time when $i = 90$ deg.**

in tether length and the current strength are made to be zero as is desired by the control law provided above.

Consider another case in which the main satellite is restricted to a polar orbit whose inclination angle equals to 90 deg. Figure 4 shows the changes of $\alpha$ a) (solid line), $\beta$ a) (dashed line), $\dot{L}$ b) and $I$ c) as functions of time. Again, the regions where $\theta$ is within the range of [75 deg, 105 deg] or [255 deg, 285 deg] are shown by the pairs of vertical dotted lines. In this case, note that it takes less time ($6 \times 10^3$ s) for the system to converge to the equilibrium position compared to Fig. 3 ($1 \times 10^4$ s), corresponding to the case when $i = 45$ deg. During the whole process, the amplitudes of $\dot{L}$ and $I$ are smaller than 0.3 m/s and 1.5 A, respectively. Again, between each pair of vertical dotted lines, $\dot{L}$ and $I$ become zero.

Incidentally, as stated above, there is a threshold value for the orbit inclination angle. When the orbit inclination angle is too small, the current strength required by the control law will become too large to be carried out in a practical situation. In other words, the control cost will become too large. The threshold value for the orbit inclination angle, however, varies for different missions, depending on several factors such as the maximum current strength the power source can provide and the maximum possible deviations in $\alpha$ and $\beta$ caused by outer perturbations. Therefore, it is necessary to perform optimization studies for the mission and system parameters according to the proposed control law.

## Conclusions

A simple and efficient closed-loop control law for electrodynamic tethers is proposed in this Note. By adjusting the rate of change in tether length and the electric current passing through the tether, the control law can assist the tether to maintain its equilibrium position in the radial direction. It is found that when the main satellite's orbit is in the equatorial plane or the orbit angle is 90 deg and 270 deg for inclined orbits, the electrodynamic force cannot provide the out-of-plane component, and this results in difficulties for the implementation of the control law. To avoid these difficulties, several restrictions are added to the original feedback control law. The applicability of the proposed control law is checked by several numerical examples. The results verify the validity of the suggested control law.

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